



Routes through Calculations

May 2021

Children's chances of maths success are maximised if they develop a deep and lasting understanding of mathematical procedures and concepts. Through Teaching for Mastery, learning is structured with great care to build deep conceptual knowledge alongside developing procedural fluency. The focus is on the development of deep structural knowledge and the ability to make connections.

This Routes through Calculations document provides information on the number facts to be learnt in KS1, the multiplication facts to be learnt in Y2, Y3 and Y4, examples of concrete materials used to represent numbers, core representations and the year groups they are used in and how each calculation looks in each year group.

This policy is based on the following ideas:

- Children should develop a clear conceptual understanding of arithmetic processes.
- Children should have fluent recall of number facts, including addition facts within 10, single digit sums and times table facts.
- Children should develop a secure understanding of efficient written algorithms for arithmetic by Year 4.
- Related operations (e.g. addition and subtraction) should be introduced together, with explicit links between them.
- The role of place value in arithmetic processes should be modelled and made clear from the start of Y2.

Fluency in number facts

+	0	1	2	3	4	5	6	7	8	9	10
0	0 + 0	0 + 1	0 + 2	0 + 3	0 + 4	0 + 5	0 + 6	0 + 7	0 + 8	0 + 9	0 + 10
1	1 + 0	1 + 1	1 + 2	1 + 3	1 + 4	1 + 5	1 + 6	1 + 7	1 + 8	1 + 9	1 + 10
2	2 + 0	2 + 1	2 + 2	2 + 3	2 + 4	2 + 5	2 + 6	2 + 7	2 + 8	2 + 9	2 + 10
3	3 + 0	3 + 1	3 + 2	3 + 3	3 + 4	3 + 5	3 + 6	3 + 7	3 + 8	3 + 9	3 + 10
4	4 + 0	4 + 1	4 + 2	4 + 3	4 + 4	4 + 5	4 + 6	4 + 7	4 + 8	4 + 9	4 + 10
5	5 + 0	5 + 1	5 + 2	5 + 3	5 + 4	5 + 5	5 + 6	5 + 7	5 + 8	5 + 9	5 + 10
6	6 + 0	6 + 1	6 + 2	6 + 3	6 + 4	6 + 5	6 + 6	6 + 7	6 + 8	6 + 9	6 + 10
7	7 + 0	7 + 1	7 + 2	7 + 3	7 + 4	7 + 5	7 + 6	7 + 7	7 + 8	7 + 9	7 + 10
8	8 + 0	8 + 1	8 + 2	8 + 3	8 + 4	8 + 5	8 + 6	8 + 7	8 + 8	8 + 9	8 + 10
9	9 + 0	9 + 1	9 + 2	9 + 3	9 + 4	9 + 5	9 + 6	9 + 7	9 + 8	9 + 9	9 + 10
10	10 + 0	10 + 1	10 + 2	10 + 3	10 + 4	10 + 5	10 + 6	10 + 7	10 + 8	10 + 9	10 + 10

Adding 1

Adding 2

Bonds to 10

Adding 0

Adding 10

Doubles

Bridging/
compensating

Near doubles

The Romero Catholic Academy

Routes through calculations

Year 1 (Within 10)	Alongside
Adding 1 (e.g. $7 + 1$ and $1 + 7$)	
Doubles of numbers to 5 (e.g. $4 + 4$)	
Adding 2 (e.g. $4 + 2$ and $2 + 4$)	Partitioning 2, 3, 4, 5, 6 and 10
Number bonds to 10 (e.g. $8 + 2$ and $2 + 8$)	
Adding 10 to a number (e.g. $5 + 10$ and $10 + 5$)	
Adding 0 to a number (e.g. $3 + 0$ and $0 + 3$)	
Near doubles (e.g. $3 + 4$ and $4 + 3$)	Partitioning 7, 8 and 9
The ones without a family! $5 + 3$, $3 + 5$, $6 + 3$, $3 + 6$	
Year 2 (Bridging 10)	
Doubles of numbers to 10 (e.g. $7 + 7$)	
Near doubles (e.g. $5 + 6$ and $6 + 5$)	Partitioning 11 – 20 into single digit addends
Bridging (e.g. $8 + 4$ and $4 + 8$)	
Compensating	

Multiplication and Division Facts




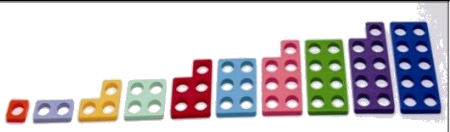
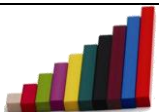
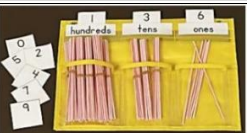
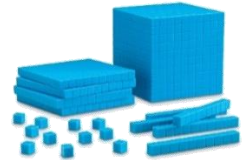


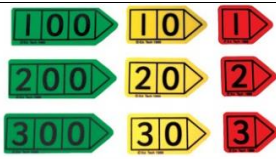

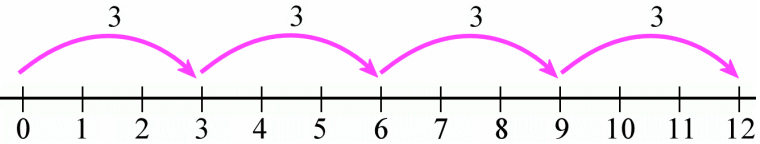
Year 2	recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables
Year 3	recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables
Year 4	recall multiplication and division facts for multiplication tables up to 12×12

We learn the tables in this order:

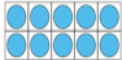

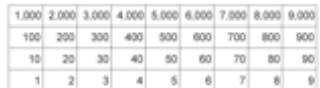
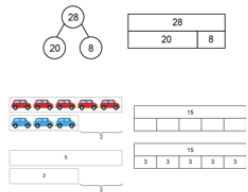
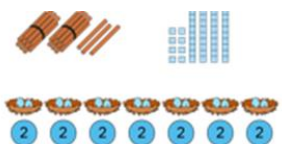
x10	x5	x2	x4	x8	x3	x6	x9	x7	x11	x12
Year 2			Year 3			Year 4				

Representing Numbers with concrete materials

The following concrete materials are used to support the children's understanding of number.

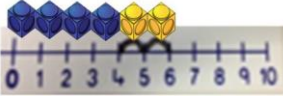
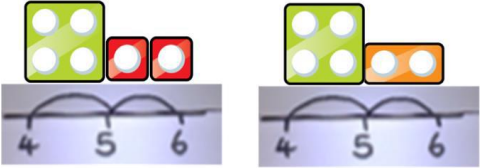
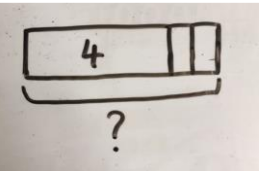
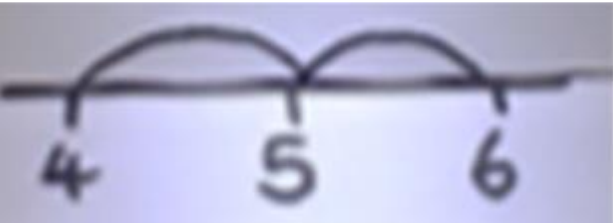
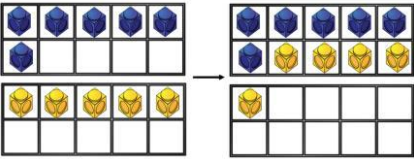
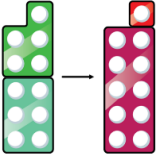
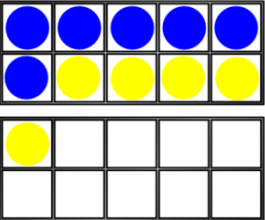
Objects		Counters	
Pegs		Numicon Shapes	
Number Rods (Cuisenaire)		Straws	
Place Value Equipment		Place Value Counters	
Number Tracks		Place Value Cards	
Bead Strings		Number Lines	

Core Representations are used to support children's understand (pictorial)

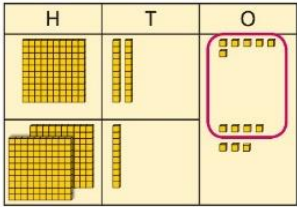
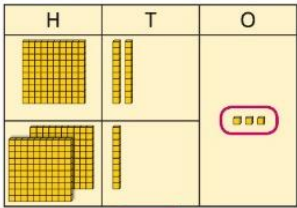
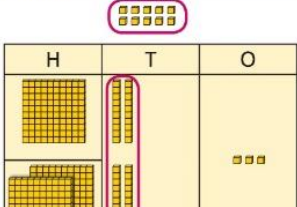
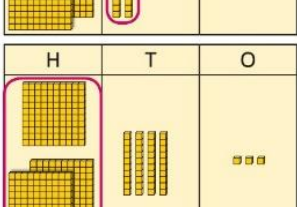
Representation		Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
	Tens Frame	Number and place value, addition and subtraction and number fluency	Addition and subtraction	Number and place value and number fluency		Number and place value, number fluency and multiplication and division	Number and place value
	Number Line	Number and place value and number fluency	Number and place value and addition and subtraction	Number and place value and fractions			
	Getegno Chart	Number and place value and number fluency			Multiplication and division	Number and place value and multiplication and division	Number and place value
	Partitioning diagrams including bar models	Addition and subtraction and number fluency	Number and place value and addition and subtraction	Number and place value, addition and subtraction and fractions	Number and place value, multiplication and division and fractions	Number and place value and fractions	Number and place value, addition and subtraction, multiplication and division and fractions
	Groups of units in addition to ones such as Diennes, PV counters		Number and place value, addition and subtraction and multiplication and division	Addition and subtraction and multiplication and division	Multiplication and division and fractions	Number and place value and multiplication and division	Number and place value

	Reception / Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Addition	Combining two parts to make a whole: part whole model. Starting at the bigger number and counting on- using cubes. Exchanging to make 10 using ten frame.	Adding three single digits. Column layout – no exchanging.	Column layout- exchanging. Using place value counters (up to 3 digits).	Column layout with exchanging. (up to 4 digits)	Column layout with exchanging. Add decimals with up to two decimal places	Column layout- exchanging. Abstract methods.
Subtraction	Taking away ones Counting back Find the difference Part whole model Make 10 using the ten frame	Counting back Find the difference Part whole model Make 10 Column layout - no exchanging	Column layout with exchanging (up to 3 digits using place value counters)	Column layout with exchanging. (up to 4 digits)	Column layout with exchanging. Subtract decimals with up to two decimal places	Column layout with exchanging. Abstract methods.
Multiplication	Recognising and making equal groups. Doubling Counting in multiples Use cubes, Numicon and other objects in the classroom	Arrays- showing commutative multiplication	Column Short multiplication (2 digit number multiplied by 1 digit)	Column short multiplication (2 and 3 digit multiplied by 1 digit)	Column long multiplication (up to 4 digit numbers multiplied by 1 or 2 digits)	Column Long multiplication (multi digit up to 4 digits by a 2-digit number)
Division	Sharing objects into groups Division as grouping e.g. I have 12 sweets and put them in groups of 3, how many groups? Use cubes and draw round 3 cubes at a time.	Division as grouping Division within arrays- linking to multiplication Repeated subtraction	Division with a remainder-using lollipop sticks, times tables facts and repeated subtraction. 2d divided by 1d using base 10 or place value counters	Short division (up to 3 digits by 1 digit- concrete and pictorial)	Short division (up to 4 digits by a 1 digit number including remainders)	Long division with place value counters (up to 4 digits by a 2 digit number) Children should exchange into the tenths and hundredths column too

Addition	Concrete & Pictorial Representations	Written Recording
<p>Reception Combining two parts to make a whole</p>	<div data-bbox="479 197 775 469"></div> <p data-bbox="439 485 1211 517">(use other resources too e.g. eggs, shells, teddy bears, cars).</p> <div data-bbox="439 517 754 762"></div> <p data-bbox="439 770 1328 842">Children to represent the cubes using dots or crosses. They could put each part on a part whole model too.</p>	<div data-bbox="1408 180 2083 292"><p>$4 + 3 = 7$ $7 = 4 + 3$ Four is a part, three is a part and the whole is seven.</p></div> <div data-bbox="1462 292 1821 722"></div>

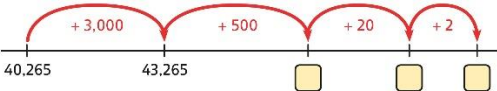
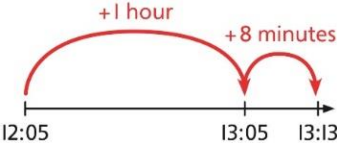
Addition	Concrete & Pictorial Representations	Written Recording
<p>Year 1</p> <p>Starting at the bigger number and counting on</p>	<p>Count on using cubes or Numicon.</p>   <p>A bar model which encourages the children to count on, rather than count all.</p> 	<p>The abstract number line:</p> <p>What is 2 more than 4?</p> <p>What is the sum of 2 and 4?</p> <p>What is the total of 4 and 2?</p> <p>$4 + 2$</p> 
<p>Year 1</p> <p>Exchanging to make 10</p>	<p>Exchange using ten frames and counters/cubes or using Numicon.</p> <p>$6 + 5$</p>   <p>Children to draw the ten frame and counters/cubes.</p> 	<p>Children to develop an understanding of equality e.g.</p> <p>$6 + \square = 11$</p> <p>$6 + 5 = 5 + \square$</p> <p>$6 + 5 = \square + 4$</p>

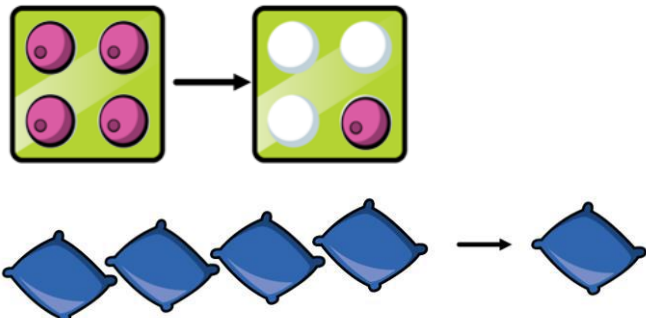
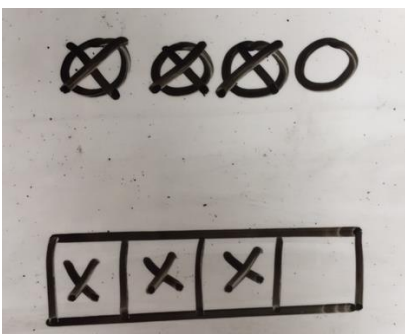
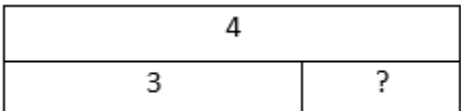
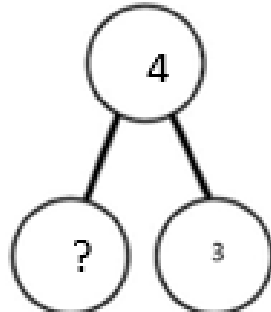
Addition	Concrete & Pictorial Representations	Written Recording
<p>Year 2</p> <p>Adding two or three single digits.</p> <p>Column layout – no exchanging.</p>	<p>TO + O using base 10. Continue to develop understanding of partitioning and place value.</p> <div data-bbox="533 268 1003 400"> </div> <p>41 + 8</p> <p>Children to represent the base 10 e.g. lines for tens and dot/crosses for ones.</p> <div data-bbox="439 499 734 735"> </div>	<p>41 + 8</p> <div data-bbox="1413 244 1592 456"> </div> <p>1 + 8 = 9 40 + 9 = 49</p> <div data-bbox="1397 587 1588 778"> </div>

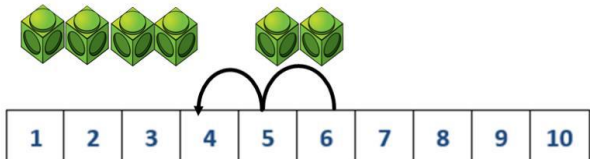
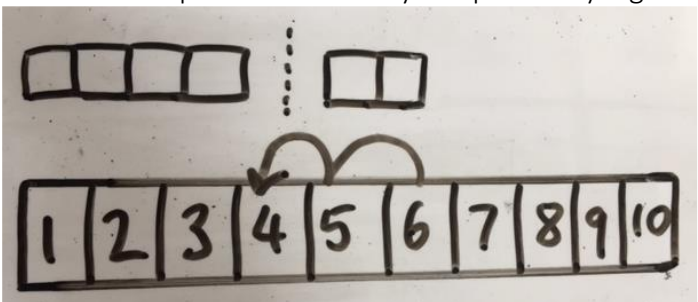

Addition	Concrete & Pictorial Representations	Written Recording
Year 3 Column layout- exchanging (up to 3 digits).	Place Value Equipment Model the stages of column addition using place value equipment on a place value grid. <div style="display: flex; flex-direction: column; align-items: center;"> <div style="display: flex; justify-content: space-around; width: 100%;"> HTO </div>  <div style="display: flex; justify-content: space-around; width: 100%;"> HTO </div>  <div style="display: flex; justify-content: space-around; width: 100%;"> HTO </div>  <div style="display: flex; justify-content: space-around; width: 100%;"> HTO </div>  </div>	<div style="display: flex; flex-direction: column; align-items: center;"> <div style="display: flex; justify-content: space-around; width: 100%;"> HTO </div> $\begin{array}{r} 126 \\ + 217 \\ \hline \end{array}$ <div style="display: flex; justify-content: space-around; width: 100%;"> HTO </div> $\begin{array}{r} 126 \\ + 217 \\ \hline 43 \end{array}$ <div style="display: flex; justify-content: space-around; width: 100%;"> HTO </div> $\begin{array}{r} 126 \\ + 217 \\ \hline 343 \end{array}$ </div>

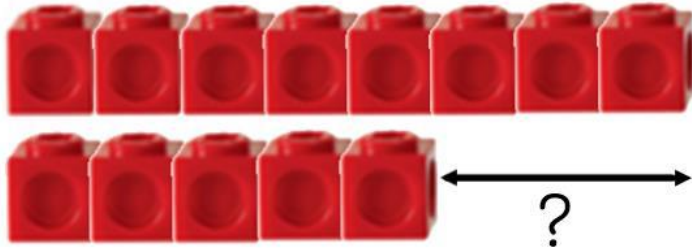
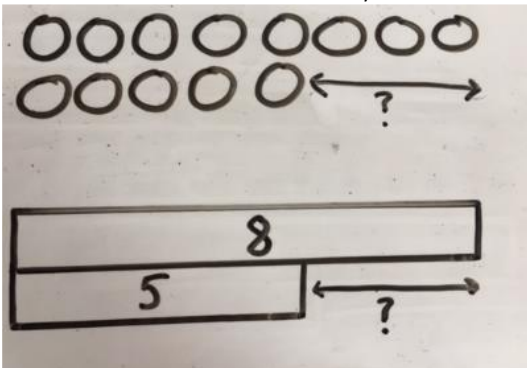
Addition	Concrete & Pictorial Representations	Written Recording
<p>Year 4</p> <p>Column layout-exchanging. (up to 4 digits)</p>	<p>Place Value Counters on Grids – Thousands, Hundreds, Tens and Units</p>	<p>Compact column addition with larger numbers</p> $ \begin{array}{r} \text{Th} \quad \text{H} \quad \text{T} \quad \text{O} \\ 1 \quad 5 \quad 5 \quad 4 \\ + 4 \quad 2 \quad 3 \quad 7 \\ \hline \end{array} $ $ \begin{array}{r} \text{Th} \quad \text{H} \quad \text{T} \quad \text{O} \\ 1 \quad 5 \quad 5 \quad 4 \\ + 4 \quad 2 \quad 3 \quad 7 \\ \hline \quad \quad 9 \quad 1 \end{array} $ $ \begin{array}{r} \text{Th} \quad \text{H} \quad \text{T} \quad \text{O} \\ 1 \quad 5 \quad 5 \quad 4 \\ + 4 \quad 2 \quad 3 \quad 7 \\ \hline \quad 7 \quad 9 \quad 1 \end{array} $ $ \begin{array}{r} \text{Th} \quad \text{H} \quad \text{T} \quad \text{O} \\ 1 \quad 5 \quad 5 \quad 4 \\ + 4 \quad 2 \quad 3 \quad 7 \\ \hline 5 \quad 7 \quad 9 \quad 1 \end{array} $

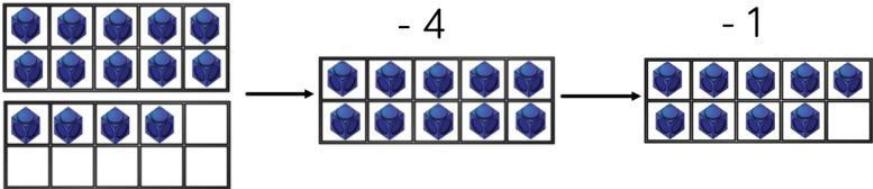
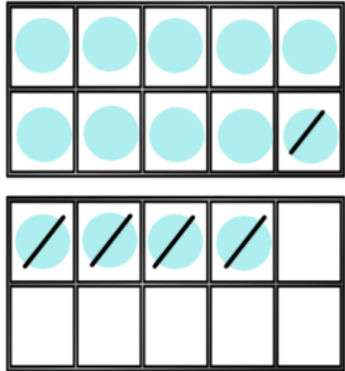
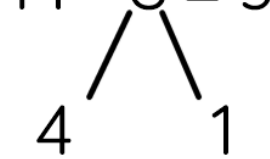
Addition	Concrete & Pictorial Representations	Written Recording																																																																																																
<p>Year 5</p> <p>Column layout-exchanging (including decimals)</p> <p>.</p>	<p>Use place value equipment on a place value grid to represent additions.</p> <p>Represent exchange where necessary.</p> <div><table><tr><th>O</th><th>•</th><th>Tth</th><th>Hth</th></tr><tr><td></td><td>•</td><td><div><div>100</div><div>100</div><div>100</div><div>100</div><div>100</div><div>100</div><div>100</div><div>100</div></div></td><td><div><div>1000</div><div>1000</div></div></td></tr><tr><td></td><td>•</td><td><div><div>100</div><div>100</div><div>100</div><div>100</div><div>100</div><div>100</div><div>100</div><div>100</div></div></td><td></td></tr><tr><td></td><td>•</td><td><div><div>100</div><div>100</div><div>100</div><div>100</div><div>100</div><div>100</div><div>100</div><div>100</div></div></td><td><div><div>1000</div><div>1000</div><div>1000</div><div>1000</div></div></td></tr></table><div><div>100</div></div></div> <div><table><tr><th>O</th><th>•</th><th>Tth</th><th>Hth</th></tr><tr><td><div><div>1000</div><div>1000</div><div>1000</div><div>1000</div><div>1000</div><div>1000</div><div>1000</div><div>1000</div></div></td><td>•</td><td></td><td></td></tr><tr><td><div><div>1000</div></div></td><td>•</td><td><div><div>100</div><div>100</div></div></td><td><div><div>1000</div><div>1000</div><div>1000</div><div>1000</div><div>1000</div><div>1000</div><div>1000</div><div>1000</div></div></td></tr></table><div><table><tr><th>O</th><th>•</th><th>Tth</th><th>Hth</th></tr><tr><td></td><td>•</td><td></td><td></td></tr><tr><td>5</td><td>•</td><td>0</td><td>0</td></tr><tr><td>+</td><td>•</td><td>1</td><td>2</td></tr><tr><td></td><td>•</td><td>6</td><td>2</td></tr></table></div></div> <p>Include examples where the numbers of decimal places are different.</p>	O	•	Tth	Hth		•	<div><div>100</div><div>100</div><div>100</div><div>100</div><div>100</div><div>100</div><div>100</div><div>100</div></div>	<div><div>1000</div><div>1000</div></div>		•	<div><div>100</div><div>100</div><div>100</div><div>100</div><div>100</div><div>100</div><div>100</div><div>100</div></div>			•	<div><div>100</div><div>100</div><div>100</div><div>100</div><div>100</div><div>100</div><div>100</div><div>100</div></div>	<div><div>1000</div><div>1000</div><div>1000</div><div>1000</div></div>	O	•	Tth	Hth	<div><div>1000</div><div>1000</div><div>1000</div><div>1000</div><div>1000</div><div>1000</div><div>1000</div><div>1000</div></div>	•			<div><div>1000</div></div>	•	<div><div>100</div><div>100</div></div>	<div><div>1000</div><div>1000</div><div>1000</div><div>1000</div><div>1000</div><div>1000</div><div>1000</div><div>1000</div></div>	O	•	Tth	Hth		•			5	•	0	0	+	•	1	2		•	6	2	<p>Add using a column method, ensuring that children understand the link with place value.</p> <div><table><tr><th>O</th><th>•</th><th>Tth</th><th>Hth</th></tr><tr><td>0</td><td>•</td><td>2</td><td>3</td></tr><tr><td>+</td><td>•</td><td>0</td><td>4</td></tr><tr><td></td><td>•</td><td>0</td><td>6</td></tr></table></div> <p>Include exchange where required, alongside an understanding of place value.</p> <div><table><tr><th>O</th><th>•</th><th>Tth</th><th>Hth</th></tr><tr><td>0</td><td>•</td><td>9</td><td>2</td></tr><tr><td>+</td><td>•</td><td>0</td><td>3</td></tr><tr><td></td><td>•</td><td>1</td><td>2</td></tr></table></div> <p>Include additions where the numbers of decimal places are different.</p> <p>$3.4 + 0.65 = ?$</p> <div><table><tr><th>O</th><th>•</th><th>Tth</th><th>Hth</th></tr><tr><td>3</td><td>•</td><td>4</td><td>0</td></tr><tr><td>+</td><td>•</td><td>0</td><td>6</td></tr><tr><td></td><td>•</td><td></td><td>5</td></tr></table></div>	O	•	Tth	Hth	0	•	2	3	+	•	0	4		•	0	6	O	•	Tth	Hth	0	•	9	2	+	•	0	3		•	1	2	O	•	Tth	Hth	3	•	4	0	+	•	0	6		•		5
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<p>Year 6</p> <p>Comparing and selecting efficient methods</p>	<p>Discuss similarities and differences between methods, and choose efficient methods based on the specific calculation.</p> <p>Compare written and mental methods alongside place value representations.</p> <div><table><tr><th>TTh</th><th>Th</th><th>H</th><th>T</th><th>O</th></tr><tr><td>40</td><td>2</td><td>6</td><td>5</td><td></td></tr><tr><td>43</td><td>2</td><td>6</td><td>5</td><td></td></tr></table><table><tr><th>TTh</th><th>Th</th><th>H</th><th>T</th><th>O</th></tr><tr><td>4</td><td>0</td><td>2</td><td>6</td><td>5</td></tr><tr><td>+</td><td>3</td><td>5</td><td>2</td><td>2</td></tr><tr><td colspan="5"><hr/></td></tr></table></div> <p>Use bar model and number line representations to model addition in problem-solving and measure contexts.</p> <div></div>	TTh	Th	H	T	O	40	2	6	5		43	2	6	5		TTh	Th	H	T	O	4	0	2	6	5	+	3	5	2	2	<hr/>					<p>Use column addition where mental methods are not efficient. Recognise common errors with column addition.</p> <p>$32,145 + 4,302 = ?$</p> <div><table><tr><th>TTh</th><th>Th</th><th>H</th><th>T</th><th>O</th></tr><tr><td>3</td><td>2</td><td>1</td><td>4</td><td>5</td></tr><tr><td>+</td><td>4</td><td>3</td><td>0</td><td>2</td></tr><tr><td colspan="5"><hr/></td></tr><tr><td>3</td><td>6</td><td>4</td><td>4</td><td>7</td></tr></table><table><tr><th>TTh</th><th>Th</th><th>H</th><th>T</th><th>O</th></tr><tr><td>3</td><td>2</td><td>1</td><td>4</td><td>5</td></tr><tr><td>+</td><td>4</td><td>3</td><td>0</td><td>2</td></tr><tr><td colspan="5"><hr/></td></tr><tr><td>7</td><td>5</td><td>1</td><td>6</td><td>5</td></tr></table></div> <p><i>Which method has been completed accurately?</i></p> <p><i>What mistake has been made?</i></p> <p>Column methods are also used for decimal additions where mental methods are not efficient.</p> <div><table><tr><th>H</th><th>T</th><th>O</th><th>·</th><th>Tth</th><th>Hth</th></tr><tr><td>1</td><td>4</td><td>0</td><td>·</td><td>0</td><td>9</td></tr><tr><td>+</td><td>4</td><td>9</td><td>·</td><td>8</td><td>9</td></tr><tr><td colspan="6"><hr/></td></tr><tr><td>1</td><td>8</td><td>9</td><td>·</td><td>9</td><td>8</td></tr></table></div>	TTh	Th	H	T	O	3	2	1	4	5	+	4	3	0	2	<hr/>					3	6	4	4	7	TTh	Th	H	T	O	3	2	1	4	5	+	4	3	0	2	<hr/>					7	5	1	6	5	H	T	O	·	Tth	Hth	1	4	0	·	0	9	+	4	9	·	8	9	<hr/>						1	8	9	·	9	8
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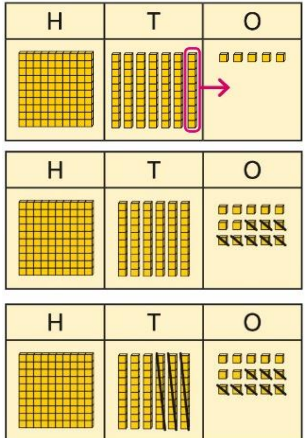
Subtraction	Concrete and Pictorial Representations	Written Recording
Reception Subtraction by taking away	<p>Physically taking away and removing objects from a whole (ten frames, Numicon, cubes and other items such as beanbags could be used).</p> <p>$4 - 3 = 1$</p>  <p>Children to draw the concrete resources they are using and cross out the correct amount. The bar model can also be used.</p> 	<p>$4 - 3 = \square$ $\square = 4 - 3$</p>  





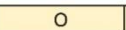
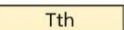


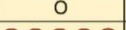
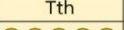











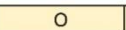
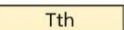


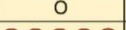
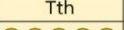











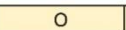
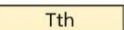


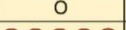
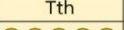







Subtraction	Concrete and Pictorial Representations	Written Recording
Reception / Year 1 Subtraction by counting back	<p>Counting back (using number lines or number tracks) children start with 6 and count back 2.</p> $6 - 2 = 4$  <p>Children to represent what they see pictorially e.g.</p> 	<p>Children to represent the calculation on a number line or number track and show their jumps. Encourage children in Year 1 to use an empty number line</p> 

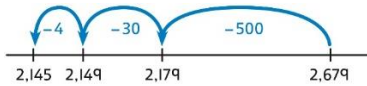
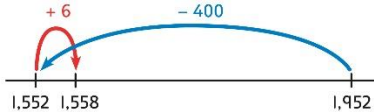
Subtraction	Concrete and Pictorial Representations	Written Recording
<p>Year 1</p> <p>Find the difference by</p>	<p>Finding the difference (using cubes, Numicon or Cuisenaire rods, other objects can also be used).</p> <p>Calculate the difference between 8 and 5.</p>  <p>Children to draw the cubes/other concrete objects which they have used or use the bar model to illustrate what they need to calculate.</p> 	<p>Find the difference between 8 and 5.</p> <p>$8 - 5$, the difference is []</p> <p>Children to explore why $9 - 6 = 8 - 5 = 7 - 4$ have the same difference.</p>

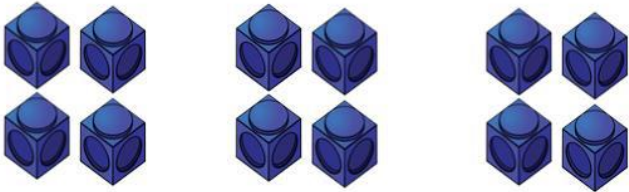
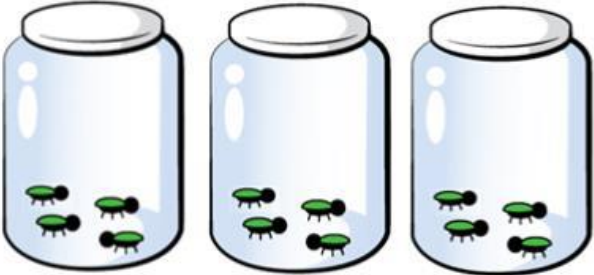
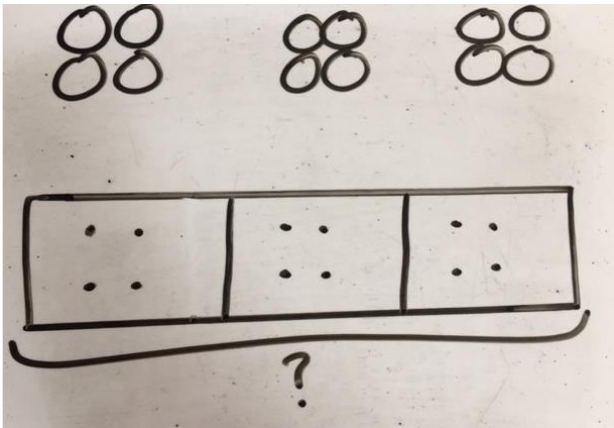
Subtraction	Concrete and Pictorial Representations	Written Recording
Year 1 Subtraction within 20	<p>Making 10 using ten frames.</p> <p>14 – 5</p>  <p>Children to represent the ten frame pictorially and discuss what they did to make 10.</p> 	<p>Children to show how they can make 10 by partitioning the subtrahend.</p> <p>14 – 5 = 9</p>  <p>14 – 4 = 10 10 – 1 = 9</p>


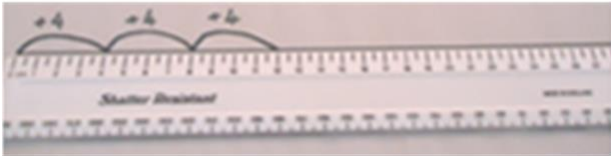
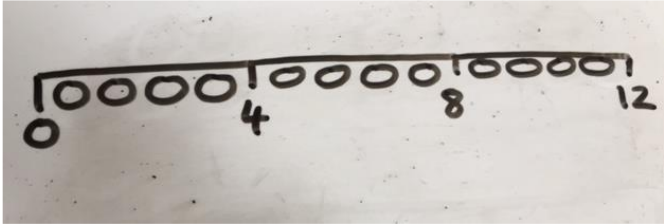
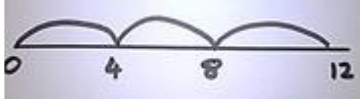
Subtraction	Concrete and Pictorial Representations	Written Recording
Year 2 Column layout – no exchanging	<p data-bbox="367 180 1182 212">Using Base 10 apparatus to take away without exchanging 48-7</p> <div data-bbox="367 252 1281 555"> </div> <p data-bbox="367 603 1106 635">Children to represent the base 10 pictorially, crossing out</p> <div data-bbox="367 675 896 1121"> </div>	<p data-bbox="1561 180 2132 244">Column layout or children could count back 7.</p> <div data-bbox="1561 252 1942 635"> </div>

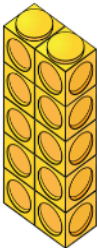
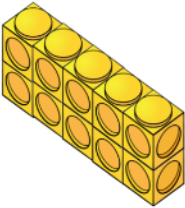
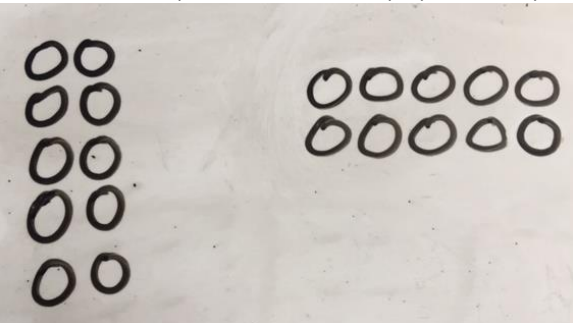
Subtraction	Concrete and Pictorial Representations	Written Recording
Year 3 Column layout with exchanging (up to 3 digits)	Place Value Equipment 	Formal column layout. Children must understand what has happened when they have crossed out digits. $ \begin{array}{r} \text{H} \quad \text{T} \quad \text{O} \\ 1 \quad 6 \quad 5 \\ - \quad 3 \quad 8 \\ \hline 1 \quad 3 \quad 7 \end{array} $ $175 - 38 = 137$
Year 4 Column subtraction with exchanging (up to 4 digits).	Place Value Counters on Grids – Thousands, Hundreds, Tens and Units. Children may need to exchange more than once.	Develop compact column subtraction with more than one exchange $ \begin{array}{r} \overset{5}{\cancel{6}} \overset{14}{\cancel{5}} \overset{12}{\cancel{3}} \overset{1}{8} \\ - \quad 2, \quad 7 \quad 8 \quad 9 \\ \hline 3, \quad 7 \quad 4 \quad 9 \end{array} $ $ \begin{array}{r} \overset{2}{\cancel{3}} \overset{16}{\cancel{7}} \overset{12}{\cancel{3}} \overset{1}{2} \\ - \quad \quad 8 \quad 3 \quad 7 \\ \hline 2, \quad 8 \quad 9 \quad 5 \end{array} $

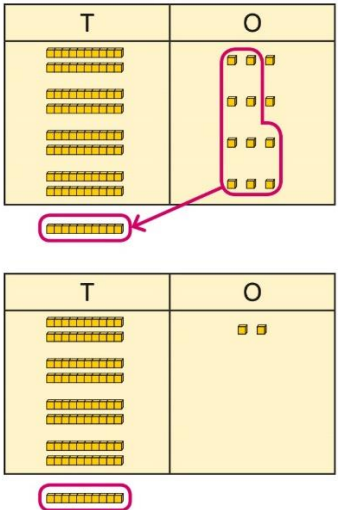
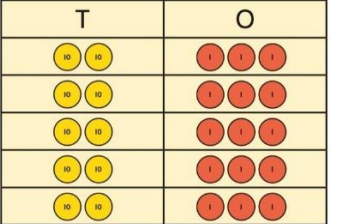
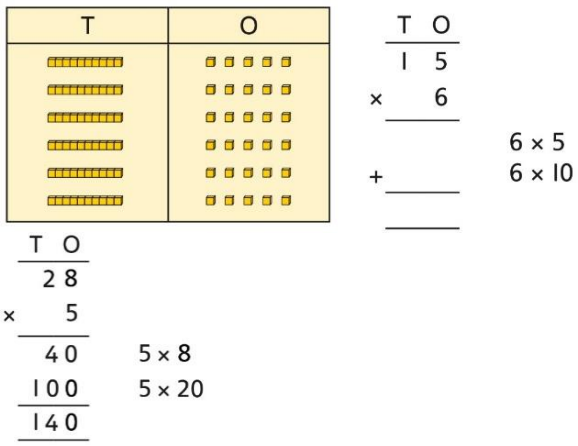
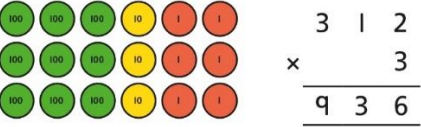
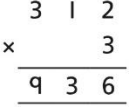
Subtraction	Concrete and Pictorial Representations	Written Recording																																																																																																													
Year 5 Column subtraction with exchanging (2-place decimals)	<p>Use a place value grid to represent the stages of column subtraction, including exchanges where required.</p> <p>$5.74 - 2.25 = ?$</p> <div><table><tr><th>O</th><th>•</th><th>Tth</th><th>Hth</th></tr><tr><td></td><td>•</td><td> </td><td></td></tr></table><p>Exchange 1 tenth for 10 hundredths.</p><table><tr><th>O</th><th>•</th><th>Tth</th><th>Hth</th></tr><tr><td></td><td>•</td><td> </td><td></td></tr></table><p>Now subtract the 5 hundredths.</p><table><tr><th>O</th><th>•</th><th>Tth</th><th>Hth</th></tr><tr><td></td><td>•</td><td> </td><td></td></tr></table><p>Now subtract the 2 tenths, then the 2 ones.</p><table><tr><th>O</th><th>•</th><th>Tth</th><th>Hth</th></tr><tr><td> </td><td>•</td><td> </td><td></td></tr></table><div><table><tr><td>O</td><td>Tth</td><td>Hth</td></tr><tr><td>5</td><td>7</td><td>4</td></tr><tr><td>-</td><td>2</td><td>2</td></tr><tr><td></td><td></td><td>5</td></tr><tr><td></td><td></td><td>.</td></tr></table><table><tr><td>O</td><td>Tth</td><td>Hth</td></tr><tr><td>5</td><td>6</td><td>14</td></tr><tr><td>-</td><td>2</td><td>2</td></tr><tr><td></td><td></td><td>5</td></tr><tr><td></td><td></td><td>.</td></tr></table><table><tr><td>O</td><td>Tth</td><td>Hth</td></tr><tr><td>5</td><td>6</td><td>14</td></tr><tr><td>-</td><td>2</td><td>2</td></tr><tr><td></td><td></td><td>9</td></tr><tr><td></td><td></td><td>.</td></tr></table><table><tr><td>O</td><td>Tth</td><td>Hth</td></tr><tr><td>5</td><td>6</td><td>14</td></tr><tr><td>-</td><td>2</td><td>2</td></tr><tr><td>3</td><td>4</td><td>9</td></tr></table></div></div>	O	•	Tth	Hth		•	 		O	•	Tth	Hth		•	 		O	•	Tth	Hth		•	 		O	•	Tth	Hth	 	•	 		O	Tth	Hth	5	7	4	-	2	2			5			.	O	Tth	Hth	5	6	14	-	2	2			5			.	O	Tth	Hth	5	6	14	-	2	2			9			.	O	Tth	Hth	5	6	14	-	2	2	3	4	9	<p>Use column subtraction, with an understanding of place value, including subtracting numbers with different numbers of decimal places.</p> <p>$3.921 - 3.75 = ?$</p> <div><table><tr><td>O</td><td>Tth</td><td>Hth</td><td>Thth</td></tr><tr><td>3</td><td>9</td><td>2</td><td>1</td></tr><tr><td>-</td><td>3</td><td>7</td><td>5</td></tr><tr><td></td><td></td><td></td><td>0</td></tr><tr><td></td><td></td><td></td><td>.</td></tr></table></div>	O	Tth	Hth	Thth	3	9	2	1	-	3	7	5				0				.
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Subtraction	Concrete and Pictorial Representations	Written Recording																																																																
<p>Year 6</p> <p>Comparing and selecting efficient methods</p>	<p>Compare subtraction methods alongside place value representations.</p> <div></div> <div><table border="1" data-bbox="369 375 844 467"><thead><tr><th>Th</th><th>H</th><th>T</th><th>O</th></tr></thead><tbody><tr><td>2</td><td>1</td><td>4</td><td>5</td></tr><tr><td>2</td><td>6</td><td>7</td><td>9</td></tr></tbody></table><div><table data-bbox="369 474 515 585"><thead><tr><th>Th</th><th>H</th><th>T</th><th>O</th></tr></thead><tbody><tr><td>2</td><td>6</td><td>7</td><td>9</td></tr><tr><td>-</td><td>5</td><td>3</td><td>4</td></tr><tr><td>2</td><td>1</td><td>4</td><td>5</td></tr></tbody></table></div></div> <p>Use a bar model to represent calculations, including ‘find the difference’ with two bars as comparison.</p> <div><div data-bbox="369 745 728 785">computer game</div><div data-bbox="369 796 728 834">puzzle book ← £12.50 →</div></div>	Th	H	T	O	2	1	4	5	2	6	7	9	Th	H	T	O	2	6	7	9	-	5	3	4	2	1	4	5	<p>Compare and select methods.</p> <p>Use column subtraction when mental methods are not efficient.</p> <p>Use two different methods for one calculation as a checking strategy.</p> <div><table data-bbox="1561 406 1713 518"><thead><tr><th>Th</th><th>H</th><th>T</th><th>O</th></tr></thead><tbody><tr><td>1</td><td>5</td><td>5</td><td>8</td></tr><tr><td>-</td><td>3</td><td>9</td><td>4</td></tr></tbody></table><div></div></div> <p>Use column subtraction for decimal problems, including in the context of measure.</p> <div><table data-bbox="1561 713 1796 841"><thead><tr><th>H</th><th>T</th><th>O</th><th>·</th><th>Tth</th><th>Hth</th></tr></thead><tbody><tr><td>3</td><td>0</td><td>9</td><td>·</td><td>6</td><td>0</td></tr><tr><td>-</td><td>2</td><td>0</td><td>·</td><td>4</td><td>0</td></tr><tr><td>1</td><td>0</td><td>3</td><td>·</td><td>2</td><td>0</td></tr></tbody></table></div>	Th	H	T	O	1	5	5	8	-	3	9	4	H	T	O	·	Tth	Hth	3	0	9	·	6	0	-	2	0	·	4	0	1	0	3	·	2	0
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Multiplication	Concrete and Pictorial Representations	Written Recording
<p>Year 1</p> <p>Recognising and making equal groups.</p> <p>Doubling</p> <p>Counting in multiples</p>	<p>Repeated grouping/repeated addition</p> <p>3×4 (3 groups of 4)</p> <p>$4 + 4 + 4$</p> <p>There are 3 equal groups, with 4 in each group.</p>   <p>Children to represent the practical resources in a picture and use a bar model.</p> 	<p>$3 \times 4 = 12$</p> <p>$4 + 4 + 4 = 12$</p>

Multiplication	Concrete and Pictorial Representations	Written Recording
Year 2	<p>Number lines to show repeated groups 3×4 (3 lots of 4)</p>   <p>Cuisenaire rods can be used too.</p> <p>Represent this pictorially alongside a number line e.g.:</p> 	<p>Abstract number line showing three jumps of four. $3 \times 4 = 12$</p> 

Multiplication	Concrete and Pictorial Representations	Written Recording
<p>Year 2</p> <p>Arrays- showing commutative multiplication</p>	<p>Use arrays to illustrate commutativity counters and other objects can also be used.</p> <p>$2 \times 5 = 5 \times 2$</p> <div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;">  <p>2 lots of 5</p> </div> <div style="text-align: center;">  <p>5 lots of 2</p> </div> </div> <p>Children to represent the arrays pictorially.</p> 	<p>Children to be able to use an array to write a range of calculations e.g.</p> <p>$10 = 2 \times 5$</p> <p>$5 \times 2 = 10$</p> <p>$2 + 2 + 2 + 2 + 2 = 10$</p> <p>$10 = 5 + 5$</p>

Multiplication	Concrete and Pictorial Representations	Written Recording
Year 3 Column layout to TU x U	<p>Using place value counters (with exchanging)</p>  <p>Children to represent the counters/base 10, pictorially e.g. the image below.</p> 	<p>Formal written method</p> 
Year 4 Column multiplication (2 and 3 digit multiplied by 1 digit)	<p>Use place value equipment alongside a column method for multiplication of up to 3-digit numbers by a single digit.</p> 	<p>Use the formal column method for up to 3-digit numbers multiplied by a single digit.</p> 

Multiplication	Concrete and Pictorial Representations	Written Recording
Year 5 Multiplying 2-digit numbers by 2-digit numbers	<p>Use an area model and add the parts.</p> <p>$28 \times 15 = ?$</p> <div><div><div><div>20 m</div><div>8 m</div></div><div><div>10 m</div><div>5 m</div></div><div><div><div>$20 \times 10 = 200 \text{ m}^2$</div><div>$20 \times 5 = 100 \text{ m}^2$</div></div><div><div>$8 \times 10 = 80 \text{ m}^2$</div><div>$8 \times 5 = 40 \text{ m}^2$</div></div></div></div><div><div>H T O</div><div>2 0 0</div><div>1 0 0</div><div>8 0</div><div>+ 4 0</div><div>4 2 0</div><div>1</div></div></div> <p>$28 \times 15 = 420$</p>	<p>Use column multiplication, ensuring understanding of place value at each stage.</p> <div><div><div>3 4</div><div>$\times 27$</div><div><div>2 3 8</div><div>6 8 0</div></div><div>34 $\times 7$</div><div>34 $\times 20$</div></div></div> <div><div>3 4</div><div>$\times 27$</div><div><div>2 3 8</div><div>6 8 0</div></div><div>34 $\times 7$</div><div>34 $\times 20$</div></div> <div><div>3 4</div><div>$\times 27$</div><div><div>2 3 8</div><div>6 8 0</div><div>9 1 8</div></div><div>34 $\times 7$</div><div>34 $\times 20$</div><div>34 $\times 27$</div></div>

Year 5

Multiplying up to 4-digits by 2-digits

Use the area model then add the parts.

	100	40	3	
10				Th H T O
2				1 0 0 0
				4 0 0
				2 0 0
				8 0
				3 0
				+
				6
				1 7 1 6

$$143 \times 12 = 1,716$$

There are 1,716 boxes of cereal in total.

$$143 \times 12 = 1,716$$

Use column multiplication, ensuring understanding of place value at each stage.

$$\begin{array}{r} 143 \\ \times 12 \\ \hline 286 \\ 1430 \\ \hline 1716 \end{array}$$

Progress to include examples that require multiple exchanges as understanding, confidence and fluency build.

$$1,274 \times 32 = ?$$

First multiply 1,274 by 2.

$$\begin{array}{r} 1274 \\ \times 32 \\ \hline 2548 \end{array}$$

Then multiply 1,274 by 30.

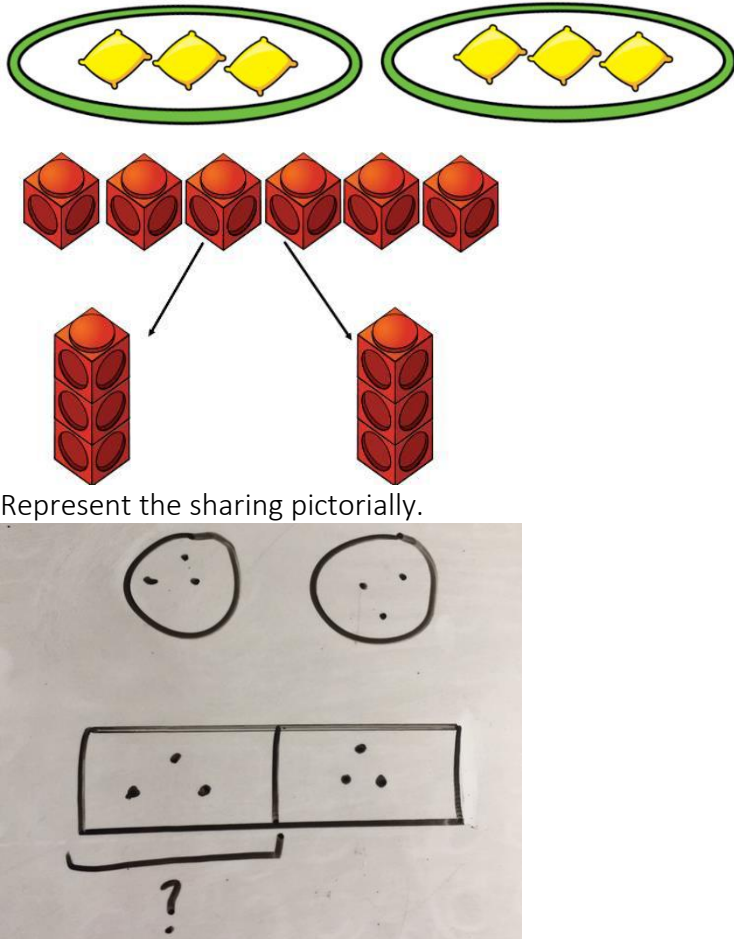
$$\begin{array}{r} 1274 \\ \times 32 \\ \hline 2548 \\ 38220 \end{array}$$

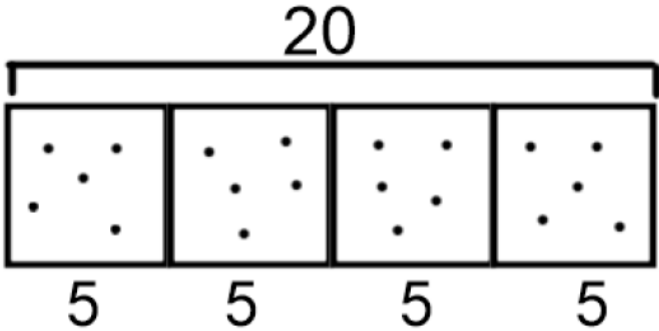
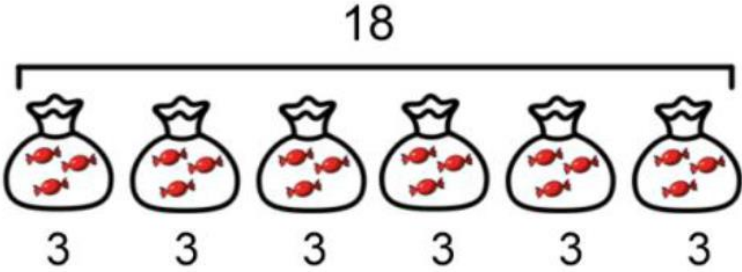
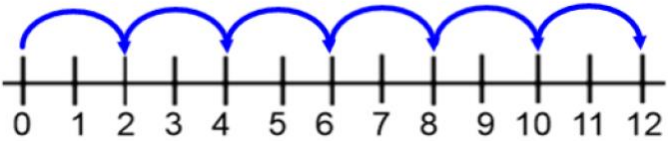
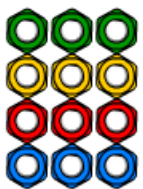
Finally, find the total.

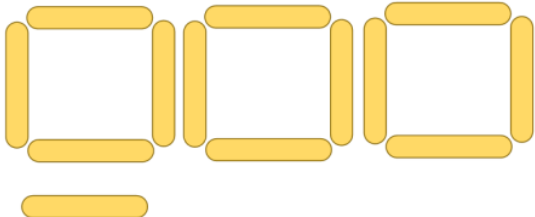
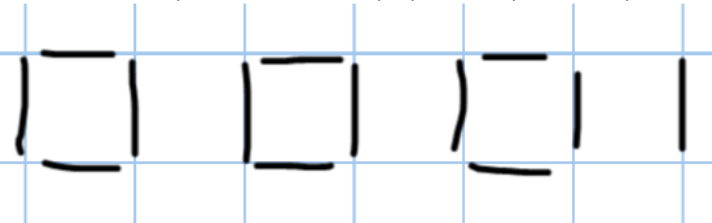


$$\begin{array}{r} 1274 \\ \times 32 \\ \hline 2548 \\ 38220 \\ \hline 40768 \end{array}$$

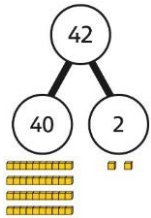
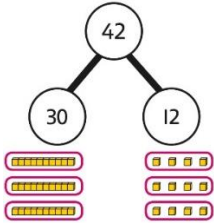
$$1,274 \times 32 = 40,768$$

Multiplication	Concrete and Pictorial Representations	Written Recording																																																				
<div>Year 6</div> <div>Multiplying Decimals</div>	<div>Represent calculations on a place value grid.</div> <div>$3 \times 3 = 9$</div> <div>$3 \times 0.3 = 0.9$</div> <div><table><tr><th>T</th><th>O</th><th>•</th><th>Tth</th></tr><tr><td></td><td></td><td></td><td><div><div>0.1</div><div>0.1</div><div>0.1</div></div></td></tr><tr><td></td><td></td><td></td><td><div><div>0.1</div><div>0.1</div><div>0.1</div></div></td></tr><tr><td></td><td></td><td></td><td><div><div>0.1</div><div>0.1</div><div>0.1</div></div></td></tr></table></div> <div>Understand the link between multiplying decimals and repeated addition.</div> <div><table><tr><th>T</th><th>O</th><th>•</th><th>Tth</th></tr><tr><td></td><td></td><td></td><td><div><div>0.2</div><div>0.2</div><div>0.2</div><div>0.2</div></div></td></tr></table><div></div></div>	T	O	•	Tth				<div><div>0.1</div><div>0.1</div><div>0.1</div></div>				<div><div>0.1</div><div>0.1</div><div>0.1</div></div>				<div><div>0.1</div><div>0.1</div><div>0.1</div></div>	T	O	•	Tth				<div><div>0.2</div><div>0.2</div><div>0.2</div><div>0.2</div></div>	<div>Use known facts to multiply decimals.</div> <div>$4 \times 3 = 12$</div> <div>$4 \times 0.3 = 1.2$</div> <div>$4 \times 0.03 = 0.12$</div> <div>$20 \times 5 = 100$</div> <div>$20 \times 0.5 = 10$</div> <div>$20 \times 0.05 = 1$</div> <div>Find families of facts from a known multiplication.</div> <div><i>I know that $18 \times 4 = 72$.</i></div> <div><i>This can help me work out:</i></div> <div>$1.8 \times 4 = ?$</div> <div>$18 \times 0.4 = ?$</div> <div>$180 \times 0.4 = ?$</div> <div>$18 \times 0.04 = ?$</div> <div>Use a place value grid to understand the effects of multiplying decimals.</div> <div><table><tr><th></th><th>H</th><th>T</th><th>O</th><th>•</th><th>Tth</th><th>Hth</th></tr><tr><td>2×3</td><td></td><td></td><td>6</td><td>•</td><td></td><td></td></tr><tr><td>0.2×3</td><td></td><td></td><td>0</td><td>•</td><td>6</td><td></td></tr><tr><td>0.02×3</td><td></td><td></td><td></td><td>•</td><td></td><td></td></tr></table></div>		H	T	O	•	Tth	Hth	2×3			6	•			0.2×3			0	•	6		0.02×3				•		
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Division	Concrete and Pictorial Representations	Written Recording		
Year 1 Sharing objects into groups Division as grouping	<p>Sharing using a range of objects. $6 \div 2$</p>  <p>Represent the sharing pictorially.</p>	$6 \div 2 = 3$ <table border="1"><tr><td>3</td><td>3</td></tr></table>	3	3
3	3			

Division	Concrete and Pictorial Representations	Written Recording
<p>Year 2</p> <p>Division as sharing</p> <p>Repeated addition</p>	<p>Sharing into equal groups</p> <p>: e.g. 20 divided by 4 is 20 shared equally into 4 parts</p>  <p>Grouping and counting</p> <p>e.g. 18 divided by 3 is 18 split into equal groups of 3</p> 	<p>Abstract number line to represent the equal groups that have been added.</p> 
<p>Year 2</p> <p>Division within arrays- linking to multiplication</p>	<p>Recognise the links between multiplication and division through use of arrays:</p> <p>$3 \times 4 = 12$</p> <p>$4 \times 3 = 12$</p> <p>$12 \div 4 = 3$</p> <p>$12 \div 3 = 4$</p> 	

Division	Concrete and Pictorial Representations	Written Recording
<p>Year 3</p> <p>Division with a remainder</p>	<p>$2d \div 1d$ with remainders using lollipop sticks. Cuisenaire rods, above a ruler can also be used.</p> <p>$13 \div 4$</p> <p>Use of lollipop sticks to form wholes- squares are made because we are dividing by 4.</p>  <p>There are 3 whole squares, with 1 left over. Children to represent the lollipop sticks pictorially.</p>  <p>There are 3 whole squares, with 1 left over. Use place value equipment to understand the concept of remainder in division.</p> <p>$29 \div 2 = ?$</p>  <p>$29 \div 2 = 14 \text{ remainder } 1$</p>	<p>65 divided by 3 equals 21 remainder 2 Children should be encouraged to use their times table facts; they could also represent repeated addition on a number line. '21 groups of 3, with 2 left over'</p> <p>$65 \div 3 =$</p>  <p>Partition to divide, understanding the remainder in context.</p> <p><i>67 children try to make 5 equal lines.</i></p> <p>$67 = 50 + 17$ $50 \div 5 = 10$</p> <p>$17 \div 5 = 3 \text{ remainder } 2$ $67 \div 5 = 13 \text{ remainder } 2$</p> <p><i>There are 13 children in each line and 2 children left out.</i></p>

Division	Concrete and Pictorial Representations	Written Recording
Year 3 2d divided by 1d	<p>Children explore which partitions support particular divisions.</p>  <p><i>I need to partition 42 differently to divide by 3.</i></p>  <p>$42 = 30 + 12$</p> <p>$42 \div 3 = 14$</p>	<p>Children partition flexibly to divide where appropriate.</p> <p>$42 \div 3 = ?$ $42 = 40 + 2$</p> <p><i>I need to partition 42 differently to divide by 3.</i></p> <p>$42 = 30 + 12$</p> <p>$30 \div 3 = 10$ $12 \div 3 = 4$</p> <p>$10 + 4 = 14$ $42 \div 3 = 14$</p>

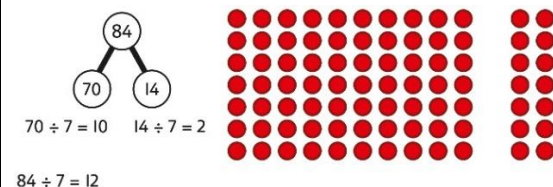
Year 4

Short division (up to 3 digits by 1 digit- concrete and pictorial)

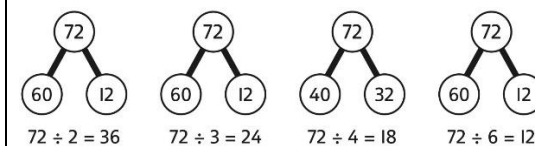
Represent how to partition flexibly where needed.

$$84 \div 7 = ?$$

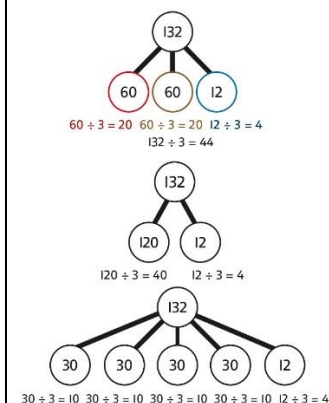
I will partition into 70 and 14 because I am dividing by 7.

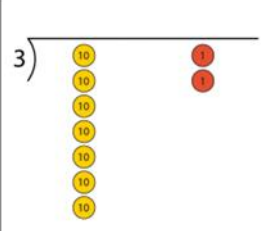
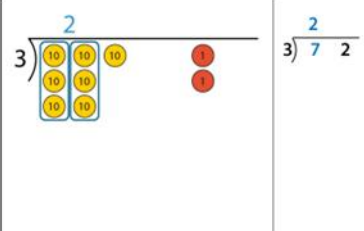
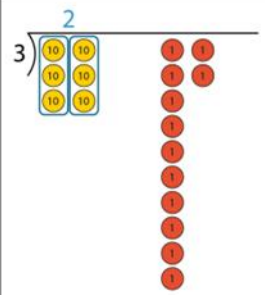
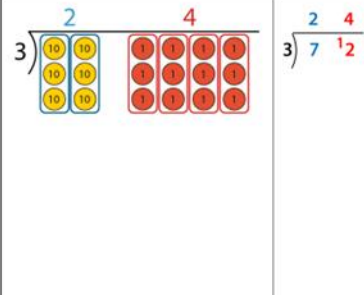
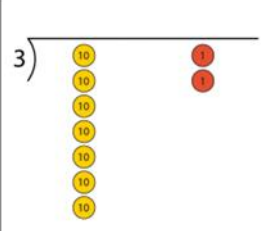
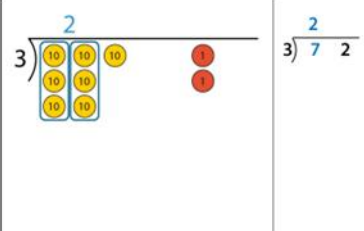
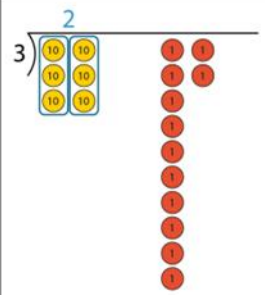
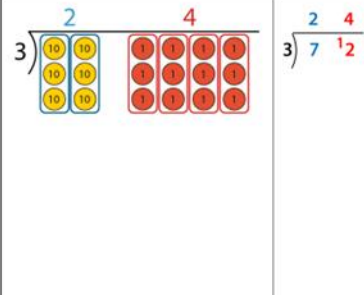
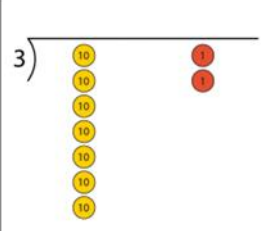
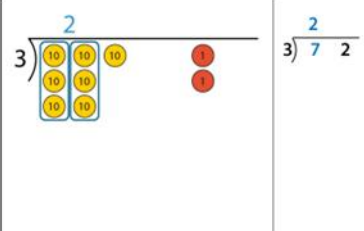
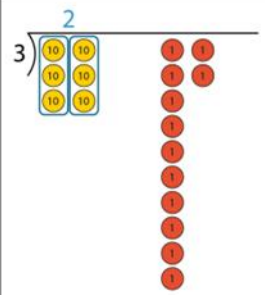
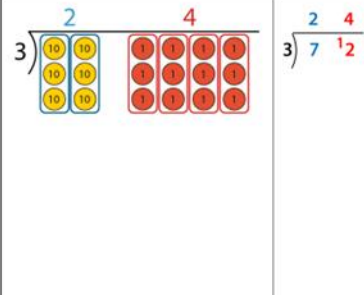


Make decisions about appropriate partitioning based on the division required.



Understand that different partitions can be used to complete the same division.



Division	Concrete and Pictorial Representations	Written Recording												
		<p><i>'Seventy-two sticks are shared equally between <u>three</u> children. How many sticks does each child get?'</i> $72 \div 3 = ?$</p> <table><tr><th>Step 1 – write the divisor and dividend</th><th>Step 2 – sharing the tens...</th></tr><tr><td></td><td></td></tr><tr><td colspan="2"><p><i>'Seventy-two divided by three.'</i></p><p>$7 \text{ tens} \div 3 = 2 \text{ tens } r 1 \text{ ten}$ <i>'Write "2" in the tens column...'</i></p></td></tr><tr><th>Step 3 – ...and exchanging</th><th>Step 4 – sharing the ones</th></tr><tr><td></td><td></td></tr><tr><td colspan="2"><p>$1 \text{ ten} = 10 \text{ ones}$ <i>'...and write "1" to the left of the ones digit of the dividend to make twelve ones.'</i></p><p>$12 \text{ ones} \div 3 = 4 \text{ ones}$ <i>'Write "4" in the ones column.'</i></p></td></tr></table>	Step 1 – write the divisor and dividend	Step 2 – sharing the tens...			<p><i>'Seventy-two divided by three.'</i></p> <p>$7 \text{ tens} \div 3 = 2 \text{ tens } r 1 \text{ ten}$ <i>'Write "2" in the tens column...'</i></p>		Step 3 – ...and exchanging	Step 4 – sharing the ones			<p>$1 \text{ ten} = 10 \text{ ones}$ <i>'...and write "1" to the left of the ones digit of the dividend to make twelve ones.'</i></p> <p>$12 \text{ ones} \div 3 = 4 \text{ ones}$ <i>'Write "4" in the ones column.'</i></p>	
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Division	Concrete and Pictorial Representations	Written Recording
<p>Year 5</p> <p>Short division (up to 4 digits by a 1 digit number including remainders)</p>	<p>Use place value equipment on a place value grid alongside short division.</p> <p>The model uses grouping.</p> <p>A sharing model can also be used, although the model would need adapting.</p> <div data-bbox="427 411 792 722"> </div> <p>Lay out the problem as a short division.</p> <p><i>There is 1 group of 4 in 4 tens.</i> <i>There are 2 groups of 4 in 8 ones.</i></p> <p>Work with divisions that require exchange.</p> <div data-bbox="427 1042 929 1444"> <p>First, lay out the problem.</p> <p>How many groups of 4 go into 9 tens?</p> <p>2 groups of 4 tens with 1 ten left over.</p> <p>Exchange the 1 ten left over for 10 ones.</p> <p>We now have 12 ones.</p> <p>How many groups of 4 go into 12 ones?</p> <p>3 groups of 4 ones.</p> </div>	<p>Use short division for up to 4-digit numbers divided by a single digit.</p> $\begin{array}{r} 0 \ 5 \ 5 \ 6 \\ 7 \overline{) 3 \ 8 \ 9 \ 2} \end{array}$ <p>$3,892 \div 7 = 556$</p> <p>Use multiplication to check.</p> <p>$556 \times 7 = ?$</p> <p>$6 \times 7 = 42$ $50 \times 7 = 350$ $500 \times 7 = 3500$</p> <p>$3,500 + 350 + 42 = 3,892$</p>

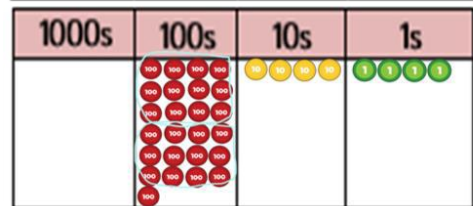
Year 6
Long division (up to 4 digits by a 2 digit number)

Long division using place value counters

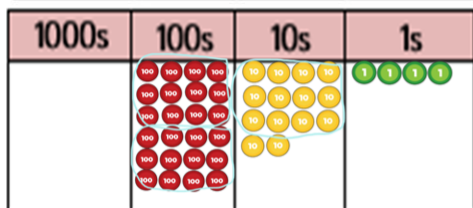
$$2544 \div 12$$



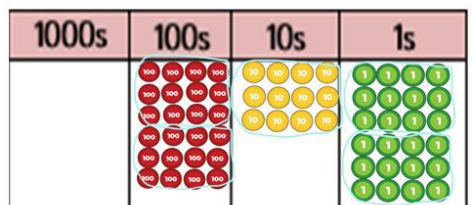
We can't group 2 thousands into groups of 12 so will exchange them.



We can group 24 hundreds into groups of 12 which leaves with 1 hundred.



After exchanging the hundred, we have 14 tens. We can group 12 tens into a group of 12, which leaves 2 tens.



After exchanging the 2 tens, we have 24 ones. We can group 24 ones into 2 groups of 12, which leaves no remainder.

$$\begin{array}{r} 02 \\ 12 \overline{) 2544} \\ \underline{24} \\ 1 \end{array}$$

$$\begin{array}{r} 021 \\ 12 \overline{) 2544} \\ \underline{24} \\ 14 \\ \underline{12} \\ 2 \end{array}$$

$$\begin{array}{r} 0212 \\ 12 \overline{) 2544} \\ \underline{24} \\ 14 \\ \underline{12} \\ 24 \\ \underline{24} \\ 0 \end{array}$$

A 0 can be used as a place holder for the tens and ones column.

Children should exchange into the tenths and hundredths column too

'Becky has 434 cm of ribbon to wrap up prizes for a school competition. Each prize needs 31 cm of ribbon. How many prizes can she wrap?'

$$434 \div 31 = ?$$

Ratio chart:

	× 31
1	31
2	62
3	
4	124
5	155
6	
7	
8	248
9	
10	310

Step 1 – write the divisor, frame and dividend

$$31 \overline{) 434}$$

Step 2 – divide the hundreds

$$\begin{array}{r} 0 \\ 31 \overline{) 434} \end{array}$$

4 hundreds \div 31 = 0 hundreds r 4 hundreds

- 'Write "0" in the hundreds column of the answer line.'

Step 3 – exchange hundreds for tens, combine with the existing tens and divide...

$$\begin{array}{r} 0 \quad 1 \\ 31 \overline{) 434} \\ \underline{31} \quad \end{array}$$

(1 ten \times 31 = 31 tens)

4 hundreds = 40 tens

40 tens + 3 tens = 43 tens

43 tens \div 31 = 1 ten and a remainder

- 'Write "1" in the tens column of the answer line and write "31" underneath the "43".'

Division	Concrete and Pictorial Representations	Written Recording
	<p>Step 4 – subtract to find the remainder</p> $\begin{array}{r} 0 \ 1 \\ 31 \overline{) 4 \ 3 \ 4} \\ \underline{3 \ 1} \\ 1 \ 2 \end{array}$ <p>(1 ten \times 31 = 31 tens)</p> <p>43 tens – 31 tens = 12 tens</p> <ul style="list-style-type: none"> • 'Write "12" underneath the "31".' 	<p>Step 5 – exchange tens for ones and combine with the existing ones</p> $\begin{array}{r} 0 \ 1 \\ 31 \overline{) 4 \ 3 \ 4} \\ \underline{3 \ 1} \downarrow \\ 1 \ 2 \ 4 \end{array}$ <p>(1 ten \times 31 = 31 tens)</p> <p>12 tens = 120 ones 120 ones + 4 ones = 124 ones</p> <ul style="list-style-type: none"> • 'Write "4" after the "12".'
	<p>Step 6 – divide the ones</p> $\begin{array}{r} 0 \ 1 \ 4 \\ 31 \overline{) 4 \ 3 \ 4} \\ \underline{3 \ 1} \\ 1 \ 2 \ 4 \\ \underline{1 \ 2 \ 4} \end{array}$ <p>(1 ten \times 31 = 31 tens)</p> <p>(4 ones \times 31 = 124 ones)</p> <p>124 ones \div 31 = 4 ones (refer to the ratio chart)</p> <ul style="list-style-type: none"> • 'Write "4" in the ones column of the answer line and write "124" underneath the "124", aligning the digits.' 	<p>Step 7 – subtract to show there is no remainder</p> $\begin{array}{r} 0 \ 1 \ 4 \\ 31 \overline{) 4 \ 3 \ 4} \\ \underline{3 \ 1} \\ 1 \ 2 \ 4 \\ \underline{1 \ 2 \ 4} \\ 0 \end{array}$ <p>(1 ten \times 31 = 31 tens)</p> <p>(4 ones \times 31 = 124 ones)</p> <p>124 ones – 124 ones = 0 ones</p> <ul style="list-style-type: none"> • 'Write "0" underneath the "31".'
	<ul style="list-style-type: none"> • 'Becky can wrap fourteen presents.' 	